An Analytical Model of Switched Reluctance Motor Based on Magnetic Field Analysis Results

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***Abstract*—A simple analytical model of switched reluctance motor (SRM) is presented in the paper. The model is developed by using two-dimensional finite element method (2D-FEM) magnetic field analysis results. Based on the flux-linkage versus current characteristics calculated for three rotor positions, aligned, unaligned and one at the midway between aligned and unaligned, named averaged, a general equation for the phase flux-linkage function of current and rotor angular position is obtained. The steady state torque versus current and rotor angular position characteristics are resulting too. The comparison between 2D-FEM calculated values and that obtained via analytical model, for two sample SRMs, stand by to sustain the model accuracy and versatility.**

***Index Terms*—flux linkage, mathematical model, switched reluctance motor**

I. INTRODUCTION

Switched reluctance motor (SRM) is a viable alternative to conventional motors like induction or synchronous, due to its simple and robust construction, wide speed range capability and reduced cost [1]. The SRM drive performances strongly depend on its designed features and control, therefore the motor's mathematical model is very important. A key factor for all SRM models is the phase flux-linkage calculation, which poses significant challenge, since both the stator and rotor have salient poles and the iron core saturation has a significant influence on the motor's operation.

The SRM's phase flux linkage calculation can be done analytically, via a numerical method, usually finite element method (FEM), or by employing a combination of analytic and numerical calculation. Many valuable works were published in the last years in this domain, such as [2,3,4] which introduce analytical models to calculate the flux linkage, [5] which develops a model based on magnetic equivalent circuits, [6,7,8] where the flux-linkage characteristics are approximated by sum of trigonometric functions or Fourier series, or quite many others with different approaches, iterating models, look-up tables based models, and so on.

The model proposed in this paper develops further the idea introduced in [9] for linear transverse flux reluctance motors and brought to a level of generality in [10] for saturated double salient reluctance motors, a category that includes the SRM.

The new representation of the SRM phase flux linkage function of rotor position and phase current takes into account the nonlinearity of the magnetic circuit via a saturation factor function. The coefficients of the referred inductance function, that takes into account the rotor position influence on the model, and of the saturation factor function, are calculated based on aligned, unaligned and averaged, situated at midway, flux linkage characteristics, obtained by 2D-FEM analysis.

The basics of the proposed SRM model, the flux-linkage model and the analytical functions parameter estimation are presented in Section II. Section III is dedicated to the SRM's electromagnetic torque. In Section IV the characteristics calculated via analytical model are compared with the ones computed via 2D-FEM analysis for two samples SRM considered. Finally, the conclusions drawn are presented in Section V.

II. MODEL BASICS AND FLUX LINKAGE CALCULATION

The proposed model is based on 2D-FEM flux-linkage versus current characteristics calculated for three rotor positions, aligned, unaligned and averaged, situated at midway from aligned to unaligned position.

The unsaturated phase flux linkage in aligned position λ_{0al} and in arbitrary rotor position λ_0 is:

$$\lambda_{0al} = L_{0al} \cdot i, \quad \lambda_0 = L_0 \cdot i \tag{1}$$

where L_{0al}, L_0 are the unsaturated values of the phase inductance in aligned and in an arbitrary rotor position.

The saturated phase flux linkages in the same positions are:

$$\lambda_{\mathrm{al}} = \lambda_{0\mathrm{al}} / k_{sal}, \quad \lambda = \lambda_0 / k_s \tag{2}$$

k_{sal}, k_s being the saturation factor function of phase current in the aligned, respectively arbitrary rotor position.

From (1), (2) comes:

$$\lambda = \lambda_0 / k_s = \lambda_{al} \frac{\lambda_0}{\lambda_{0al}} \frac{k_{sal}}{k_s} = \lambda_{al} \cdot l_{0r} / k_{sr} \tag{3}$$

$$l_{0r} = \frac{L_0}{L_{0al}}, k_{sr} = \frac{k_s}{k_{sal}} \quad (4)$$

The unsaturated inductance ratio l_{0r} does not depend on current, while the saturation factor function ratio k_{sr} depends on both current and rotor position. Their estimation is done by using simple functions which contain both a cosinusoidal term

$$l_{0r}(\theta)=a_l+b_l\cos\theta \quad (5)$$

$$k_{sr}(i,\theta)=a_s(i)+b_s(i)\cos\theta \quad (6)$$

Three values of unsaturated phase inductance are necessary to obtain the referred inductance function $l_{0r}(\theta)$, aligned L_{0al}, average L_{0av} and unaligned L_{0un}. Three points of $l_{0r}(\theta)$ characteristics are known then:

$$l_{0un} = \frac{L_{0un}}{L_{0al}}, l_{0av} = \frac{L_{0av}}{L_{0al}}, l_{0al} = 1 \quad (7)$$

and the $l_{0r}(\theta)$ characteristic coefficients a_l and b_l can be calculated through a curve fitting procedure.

By increasing the number of flux linkage characteristics used, the $l_{0r}(\theta)$ approximation is a bit different, but the differences are not that important, as one can see from Fig. 1, where two approximations are given for the same sample SRM, S1. One is estimated using only three characteristics, aligned, averaged and unaligned, l_{0r3}, and the other one is obtained by using eleven flux linkage characteristics from unaligned to aligned position, l_{0r11}.

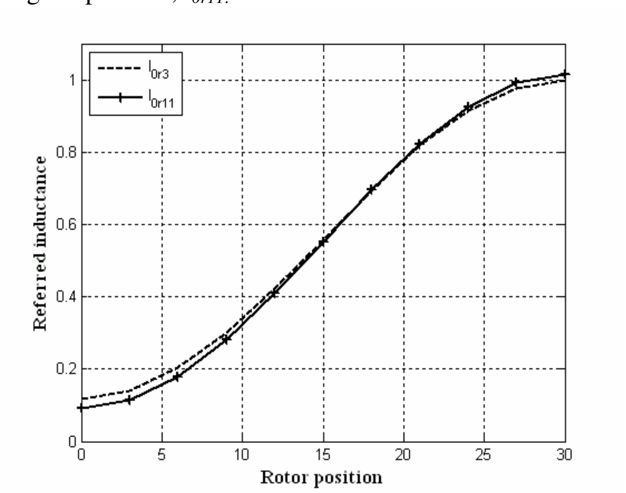

Fig. 1 Referred inductance functions l_{0r3}, l_{0r11} versus rotor position

The saturation function k_{sr} is calculated through a similar procedure in two steps.

First, a saturation function depending on the rotor position is calculated for each considered flux linkage characteristic, at constant current, resulting a set of saturation functions for different currents, i_1, i_2, …,:

$$\begin{aligned} k_{sr1} &= a_{s1} + b_{s1}\cos\theta \\ k_{sr2} &= a_{s2} + b_{s2}\cos\theta \\ &\ldots\ldots \end{aligned} \quad (8)$$

Finally, in the saturation function $k_{sr}(i,\theta)$, (6), the coefficients $a_s(i)$ and $b_s(i)$, function of current, are polynomial estimation from already existing values a_{s1}, a_{s2}…and b_{s1}, b_{s2}…respectively.

The calculation procedure for the saturation functions at different currents, in the case of a sample 8/6 SRM, S1, is illustrated in Table 1, where i_r represents the phase rated current value.

TABLE I. SATURATION FUNCTION FOR DIFFERENT CURRENT VALUES

Current	k_{sral}	k_{srav}	k_{srun}	$k_{sr}(\theta)$, i=ct
1.2*i_r	1	0.863	0.652	$0.82 + 0.173\cos\theta$
i_r	1	0.902	0.751	$0.87 + 0.124\cos\theta$
0.8*i_r	1	0.933	0.874	$0.936 + 0.063\cos\theta$
0.6*i_r	1	0.964	0.946	$0.973 + 0.027\cos\theta$

The resulting polynomial estimated saturation coefficients $a_s(i)$ and $b_s(i)$ of the saturation function $k_{sr}(i,\theta)$, obtained via a curve fitting procedure, by using the elemental values given in Table 1, are:

$$a_s(i)=-0.0006387i^2+0.0437i-0.2681 \quad (9)$$

$$b_s(i)=-0.001387i^2+0.003325i+1.01 \quad (10)$$

In Fig. 2, the variation of the saturation functions, given in Table 1, versus rotor position at different current values is presented.

The quality of the curve fitting procedure applied to determine the particular saturation function and the coefficients $a_s(i)$ and $b_s(i)$ of the general saturation function is important, the overall model accuracy depending on it.

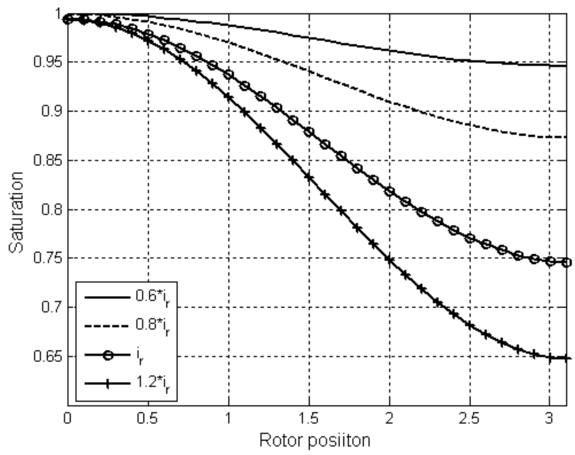

Fig. 2 Saturation functions versus rotor position

To estimate, via a curve fitting procedure, the equation describing the aligned flux linkage characteristic function of the phase current values, a ratio of polynomials was employed:

$$\lambda_{al} = i/(ai^2 + bi + c) \quad (11)$$

III. ELECTROMAGNETIC TORQUE

The SRM's electromagnetic torque should be calculated with the usual equation:

$$T = \int_0^i \frac{\partial \lambda(i,\theta)}{\partial \alpha} di, \ \theta = Q_R \cdot \alpha \quad (12)$$

where Q_R represents the rotor number of poles.

The electromagnetic torque final equation is:

$$T = Q_R \sin\theta \int_0^i \frac{a_l b_s(i) - b_l a_s(i)}{[a_s(i) + b_s(i)\cos\theta]^2} \frac{i \cdot di}{ai^2 + bi + c} \quad (13)$$

An analytic integration of (13) is possible, but a complicated sum of functions results. Therefore, an approximation based on a particular case of the Newton-Cotes method with the interpolating Lagrange polynomial of first degree [11] is proposed in the paper. Then, for a certain phase current i_{ph}, and for each rotor angular displacement value θ_j considered, the torque results:

$$T_{\theta_j} = Q_R \sin\theta_j \cdot \Delta i (f(\Delta i) + f(2\Delta i) + ... + f(i_{ph})/2) \quad (14)$$

where θ_j is a given angle, Δi is the increment of current, and f is the function to be integrated (13).

If n is the number of current sub domains considered then ◘$i = i_{ph}/n$ and the f function is zero when $i = 0$.

IV. CALCULATED RESULTS

Two sample SRMs were considered S1 and S2. S1 is a 8/6 poles SRM with four stator phases, while S2 is a 6/4 poles machine with three phases. The main data and dimension values for the two sample motors are g=.5mm, l_{st}=66mm, D_i =200mm, i_{phr}=10A, N_c=93 for S1 and respectively g=.3mm, l_{st}=142mm, D_i =166.7mm, i_{phr}=50A, N_c=20 for S2, where g is the air-gap length in aligned position, l_{st} the axial stack length, D_i the stator interior diameter, N_c the number of turns per pole coil, and i_{phr} the rated phase current.

In the S1 case the estimations for l_{0r} and λ_{al} are:

$$l_{0r}(\theta) = 0.565 + 0.441\cos\theta \quad (15)$$

$$\lambda_{al} = i_r / (0.3386 \cdot i_r^2 - 2.626 \cdot i_r + 45.55) \quad (16)$$

while the saturation function polynomial coefficients are given by (9), (10).

In Fig. 3, a comparison between the flux-linkage versus phase current characteristics, calculated via 2D-FEM respectively by using the proposed analytical model is given in the case of S1 sample SRM.

As one can see from Fig. 3, the errors are usually very small, and they do not alterate the over all model accuracy.

In the case of SRM sample S2, a motor supplied with 12V dc, the flux linkage characteristics analytical estimation is:

$$\lambda(i,\theta) = \frac{i}{0.213i^2 - 5.5529i + 819.7} * \frac{0.5639 + 0.4376\cos(\theta)}{a_s(i) + b_s(i)\cos(\theta)} \quad (17)$$

$$a_s(i) = -4.46 \cdot 10^{-5} i^2 - 79.59 \cdot 10^{-5} i + 1.019 \quad (18)$$

$$b_s(i) = 5.194 \cdot 10^{-5} i^2 + 7.749 \cdot 10^{-5} i + 0.01184 \quad (19)$$

The comparison between flux linkage versus current characteristics at different rotor position obtained via 2D-FEM and calculated by using the analytical estimation is given in Fig. 4 for the sample SRM, S2. The model accuracy is good in this case too.

The torque versus rotor position at different currents, the rated current characteristic being the upper one, for the S2 sample SRM were calculated by employing the procedure presented in Section III and the results are given in Fig. 5 against the one obtained via 2D-FEM analysis, for the same currents.

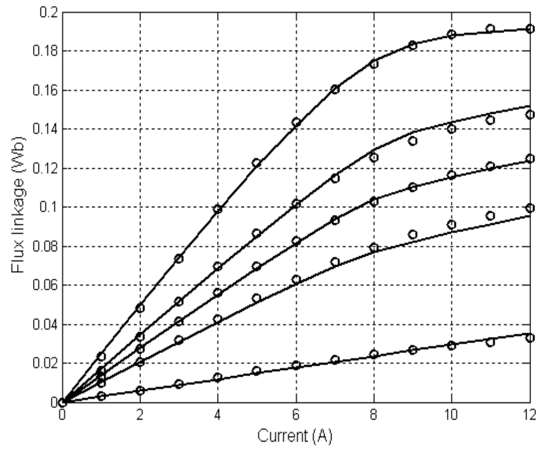

Fig. 3 Phase flux linkage versus phase current at different rotor positions, S1

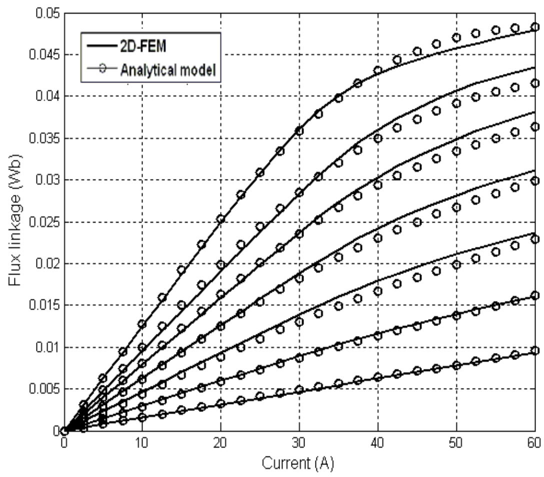

Fig. 4 Phase flux linkage versus phase current at different rotor position, S2

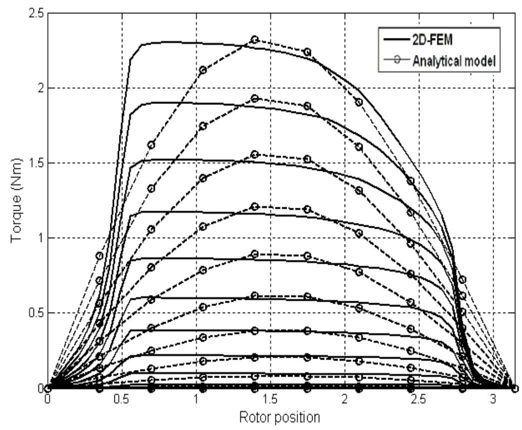

Fig. 5 Torque versus rotor position at rated current, S2

V. CONCLUSION

A simple analytical model of SRM, developed by using 2D-FEM analysis results, is presented in the paper. The model contains three analytical functions for estimating the phase flux linkage versus current characteristics:

i) One for the phase flux linkage versus current characteristic calculated via 2D-FEM at aligned rotor position
ii) A function for the variation of the unsaturated inductance ratio versus rotor position
iii) A saturation factor function which depends on both phase current and rotor position and considers the influence of the variable saturation on the phase flux linkage.

The unsaturated inductance ratio function and the saturation factor function are obtained based on three phase flux linkage versus current characteristics, calculated via 2D-FEM for the aligned, averaged and unaligned rotor positions.

A simple procedure to calculate the electromagnetic torque based on phase flux linkage analytical estimation is developed too. The torque calculation is done by employing the Newton-Coates method.

Two sample SRMs were designed, one being under construction, and their phase flux-linkage characteristics were calculated via 2D-FEM analysis. The proposed analytical model was applied on both sample SRMs and the results were found in good agreement with the ones calculated in the designing stage.

The proposed model gives quite accurate results having three important features:

i) It is simple and can be obtained easy with only few calculations
ii) It evinces clearly the influence of the rotor position via the referred unsaturated inductance function and of the core saturation at different rotor position, via saturation factor
iii) It can be extended to any other double side slotted machine with minor changes.

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